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A Single Radio Based Channel Datarate-aware Parallel Rendezvous MAC Protocol for Cognitive Radio Networks

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***Abstract*—Channel hopping based parallel rendezvous multi-channel MAC protocols have several advantages since they do not need a control channel, require only one transceiver, and produce higher system capacity. However, channel hopping sequences in existing parallel rendezvous MAC protocols have been designed as irrelevant to channel datarates, leading to under-utilization of channel resources in multi-rate multi-channel networks. Considering that datarates among channels may be different, we propose a dynamic parallel rendezvous multi-channel MAC protocol for synchronized cognitive radio networks in which the secondary users adjust their own distinct hopping sequences according to the datarates of the available channels, in a datarate-aware manner. A Markov chain based model has been developed to analyze the aggregate datarate of the proposed protocol. Numerical results show that the proposed method can improve the aggregate datarate significantly, compared with that of the existing parallel rendezvous MAC protocol.**

***Index Terms*—Cognitive radio networks, MAC protocol, Parallel rendezvous, Datarate-aware.**

I. Introduction

Spectrum is one of the most valuable resources in wireless communications. With increased popularity of wireless products and applications, the unlicensed bands such as Industrial, Scientific and Medical (ISM) have become over-crowded. On the other hand, a large portion of the assigned spectrum is sporadically used and a significant amount of the spectrum remains under-utilized. Cognitive Radio (CR) [1], as a promising solution to utilize the unused spectrum, has become a hot research topic these days.

However, the functions of CR devices become very limited if they do not form a network. Together with existing legacy infrastructure and/or ad hoc networking devices, cognitive radios can form a Cognitive Radio Network (CRN). This new type of network is built based on cognitive radio terminals and wireless networking technologies, and can transport packets based on cognitive radio links to facilitate emerging services and applications.

To form a CRN, Media Access Control (MAC) protocols are of great importance, especially for multi-channel CRNs. Existing multi-channel CRN MAC protocols can be classified into two categories: single rendezvous or parallel rendezvous. Single rendezvous MAC protocols [2-6] have a control channel as the rendezvous channel, and nodes can exchange all control information and negotiate parameters for data transmission on this channel. This control channel, however, can become a bottleneck under information exchange operations [7] and some such MAC protocols also need an additional transceiver which is always tuned onto the control channel, e.g. [5]. Parallel rendezvous MAC protocols, on the other hand, do not need a common control channel. The basic idea of parallel rendezvous protocols is that nodes jump among different channels according to their own sequences and the control information is exchanged at different channels when nodes meet. It has been demonstrated that parallel rendezvous MAC protocols, like Multi-channel MAC (McMAC) [8] and Slotted Seeded Channel Hopping (SSCH) [9] in a multi-channel wireless networks, generally outperform single rendezvous MAC protocols [7]. These protocols do not have bottleneck like in single rendezvous MAC protocols and they are all based on a single transceiver. The authors of [10] extended McMAC from general multi-channel networks into CRNs.

In the proposed MAC protocol in [10], the authors have only considered the situation where the channels are balanced, i.e., with identical datarate for all users, and the channel hopping sequences used by Secondary Users (SUs) in [10] are statistically uniform distributed. We argue, however, that in cognitive radio networks, the channels for Primary Users (PUs) may have different maximum transmission power limits and the bandwidth which SUs can utilize may be different. That is, each of these channels can have different datarates available for SUs [11]. If the datarates on different channels are not the same, it would be advantageous to introduce a method which can adjust communications according to the datarates of these channels. In this paper, we propose a channel hopping based parallel rendezvous MAC protocol for synchronized CRN with adaptive hopping sequence for unbalanced channel datarates. The main idea of our protocol is to adjust the hopping sequence of SUs according to the datarates available for SUs in different channels, so that better channel utilization can be achieved. For comparison convenience, we refer to this method as datarate-aware MAC (DRA-MAC) protocol while the method proposed in [10] is referred to as channel datarate-independent MAC (DRI-MAC).

The rest of this paper is organized as follows. Section II describes the channel model and assumptions. Section III presents the proposed MAC protocol while its performance analysis based on a Markov chain model is carried out in Section IV. In Section V, the numerical results and comparison with DRI-MAC are given. Finally, the paper is concluded in Section VI.

II. Channel Model and Assumptions

Assume that each SU in a CRN is equipped with only one transceiver and SUs cannot transmit and receive messages at the same time. The transceivers of SUs are Software Designed Radio (SDR)-based so that they can dynamically use the channels when these channels are not occupied by PUs. Figure 1 below illustrates the channel occupancy of PUs in a channel.

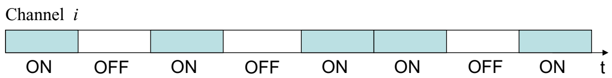

Fig. 1. Channel occupancy of channel i by PUs.

Assume that there are G channels and each channel assigned to PUs follows independent and identically distributed (i.i.d) ON/OFF random process. The ON period means that the channel is occupied by a PU and the OFF period presents that the channel is idle. The same as in [5], each licensed channel is considered to be time-slotted such that the PUs communicate with each other in a synchronized manner. The length of each time slot is equal. SUs, which are also synchronized with the PUs, opportunistically access the licensed spectrum when it is available [5].

Let α_i be the probability that the ith channel transits from state ON to state OFF and β_i be the probability that the ith channel transits from state OFF to state ON, where $1 \leq i \leq G$. Then the channel state can be modeled as a simple two-state Markov chain [5], [10] as shown in Figure 2. Then the availability of the ith channel for SUs, denoted by γ_i, which is the state probability of the corresponding Markov chain of being OFF, i.e., the channel is not occupied by PUs, can be expressed as $\gamma_i = \alpha_i/(\beta_i + \alpha_i)$, $1 \leq i \leq G$.

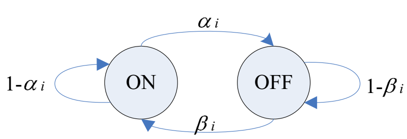

Fig. 2. ON/OFF channel model.

For the ON/OFF channel model, assume further that each SU can sense precisely the signal of PUs that it receives in each particular channel it tunes onto. The envisaged scenario for this investigation is that SUs gather in a limited geographic area while the coverage and distance scale of PUs is far larger than that of SUs', hence the SUs are covered by the same set of PU systems. This implies that the results of channel sensing by each SU node in a particular channel is the same for all SUs. It is assumed that all the SUs are in close enough proximity to be able to communicate with each other. We do not consider the mobility of SUs.

III. MAC protocol description

In this section, the principle of DRI-MAC is presented firstly, followed by a detailed description of the proposed DRA-MAC.

A. DRI parallel rendezvous MAC protocol

According to [10], each SU has its own pseudo-random hopping sequence which is uniformly distributed [12] and switches across available channels with *equal probability*. SUs decide their own hopping sequences based on their own MAC addresses using the same hopping sequence generating algorithm. The hopping sequence is fixed for a given SU. Since the MAC address of nodes is a necessity and the sequence generating algorithm is the same, the overhead of broadcasting the hopping sequence is reduced. Each SU periodically broadcasts beacons with its own hopping information over an unused channel. Once a sender receives the hopping sequence of a receiver, it can follow the receiver's hopping sequence and meet it if the sender has packets to this intended receiver. It is also possible for a potential transmitter to ask other nodes for the intended receiver's hopping sequence.

In order to check the channel state, a quiet period is introduced in the beginning of *each* slot. During this period, every SU in different channels keeps silence and listens to the channel to check if there is a PU transmission. If PUs are not there, SUs deem that it is proper to use the channel. Figure 3 illustrates the principle of the DRI-MAC operation. A basic feature of the DRI-MAC is the equal access chance for all channels regardless of different datarates. Therefore, an enhanced datarate-aware hopping scheme would help to better utilize channel resources.

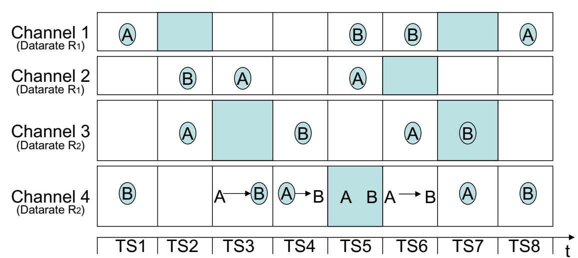

Fig. 3. Illustration of DRI-MAC. The highlighted slots mean that the time slots are used by PUs. A, B are SUs. TS1-8 mean time slots respectively. A, B with a circle denote their predefined hoping pattern. A and B are in Channel 1 and Channel 4 respectively in TS1 and will jump to Channel 3 and Channel 2 at TS2. In TS3, A would jump to Channel 2 if it has no packets to send. As A has data to send to B, A follows B's sequence and jumps to Channel 4 in TS3 instead of jumping to Channel 2. They will stay in the same channel till the transmission is finished (as in TS3-TS6). During the transmission period, if a PU comes out (as in TS5), SUs will wait until the next slot and then transmit if the channel is idle (as in TS6). The height of each channel corresponds to datarate, which indicates two different datarates exist in this illustration.

B. The proposed DRA-MAC protocol

Like other multi-channel rendezvous MAC protocols, the proposed MAC protocol does not need a control channel. The channel sensing and data transmission strategies of DRA-MAC are similar to that of DRI-MAC. The difference is, however, that the hopping pattern is designed according to channel datarates in our case. In what follows, we will first describe the basic channel hopping sequences and then explain the enhanced datarate-aware hopping sequence.

1) Basic hopping sequence: We adopt the hopping sequence generation method that is used in McMAC [8] to generate basic sequences which are uniformly distributed. The length of the sequence should be 10 times longer than the number of channels.

2) Datarate-aware hopping sequence: The datarate-aware hopping sequence is based on the basic hopping sequence with necessary modifications. More specifically, a portion of the basic sequence needs to be adjusted according to channel datarates, while the rest of the sequence is still kept the same as the pre-defined basic sequence. For example, there are 2 channels that offer different datarate for SUs. The datarate in Channel 1, R_1 is higher than that in Channel 2, R_2. Suppose a snapshot of a node's basic hopping sequence is [1, 2, 2, 1, 2, 1, 1, 2], which means that initially SUs will jump evenly between Channel 1 and Channel 2. Since $R_1 > R_2$, more hops will be preferred to be allocated in Channel 1, according to DRA-MAC. The resulted sequence could then look like [1, 2, 1, 1, 2, 1, 1, 1], which leads to higher chance for channel access of Channel 1.

However, the adjustment must be carefully designed to avoid the problem of co-behaviors which means that most SUs may jump into the same channel which has the highest datarate. This undesired problem may induce not only congestion in that channel and degradation to throughput, but also waste the utilization of other available channels. To avoid this problem, the following method is adopted.

Assume that the datarate for SUs in the ith channel is R_i, where $1 \leq i \leq G$. Let SU(i) be the SU that jumps into the ith channel according to its basic hopping sequence in its next hop. Let $\overline{R} = \sum_{j=1}^{G} R_j/G$ and $A = \{Channel\ j|R_j > \overline{R}, j = 1 \cdots G\}$. The adjustment algorithm works as follows:

1. If $R_i \geq \overline{R}$, SU(i)s which plan to jump in channel i will remain in the basic hop and will not deviate from channel i.
2. Else
(1) With probability $R_i/\overline{R}$, SU(i)s which plan to jump into channel i will remain in the basic hop and will not deviate from channel i.
(2)With probability $(1 - R_i/\overline{R}) \cdot (R_j - \overline{R})/\sum_{k \in A}(R_k - \overline{R})$, SU($i$)s will select channel j, $j \in A$.

With this scheme, SU nodes will jump according to datarates, and at the same time, avoid the co-behavior problem. The proof of this statement will be given in the appendix.

3) Beacon advertisement: Once a node adjusts its basic sequence, it must let other nodes know its new sequence. Otherwise other nodes which have data to transmit cannot find it. This message can be delivered in the following way. An SU generates the basic hopping sequence first. Based on datarate information and the basic hopping sequence, it can make a decision on which hops need to be adjusted according to the above algorithm. It can then inform others by adding the adjustment results in the periodical beacons.

Because there is no control channel, the beacon message cannot be received by SUs that are not in the current beacon-sender's channel. In order to let most SUs receive the beacon message earlier, when SUs receive another SU's beacon information with hopping adjustment, they will rebroadcast it once in the next time slot to make it known by more nodes. This procedure will help disseminating beacon messages faster than broadcasting by the SU itself, channel by channel, because the SUs which overheard the message will distribute this information into other channels in the next time slot. If an SU overhears the same beacon information from another node in the next time slot before its rebroadcast, it will not forward the beacon information again. Since the beacon information is forwarded only once, there is a probability that the message is not rebroadcast in all these channels within two time slots. Then, SUs can ask the others for a list of their known SUs about the hopping sequences. For this purpose, it can broadcast an inquiring message and other nodes which have that information can response to it. SUs that overhear this message can update their own hopping sequence information.

4) Data flow transmission: Each SU keeps a queue for each destination to avoid head-of-line blocking [8]. In each slot, if it is not occupied by a PU, SUs can negotiate for data transmission. Negotiation is needed because an intended receiver may be in another channel as a transmitter. Therefore there is a risk of packet loss if data is transmitted directly. Without negotiation, furthermore, it is possible that two or more transmitters hop in to the same channel for data transmission, resulting in collision. Negotiation which is done after the quiet period, can avoid such potential collisions. When negotiations are successfully done, the data transmission can be carried out.

If two SUs cannot finish the transmission within a time slot, they will continue using the same channel for data exchange in the next time slot, which escapes the switching penalty. An ongoing transmission between two SUs may be interrupted by sudden channel occupancy of PUs if it needs more than one time slot. In this case, the communicating pairs will pause and hold transmission if the channel is occupied by any PUs again during their data transmission. In order to guarantee that the not-yet-finished transmission has the highest priority, the unfinished transmission can start immediately after the quiet period while new transmitters will sense the channel after the quiet period and negotiate for transmission.

IV. Aggregate Datarate Analysis

In this section, we analyze aggregate datarate of the proposed protocol. Aggregate datarate means the achieved amount of bits per second the SUs in this system can obtain, considering injected traffic load into the system and each specific value of channel availability. We assume that channel availability γ_i is the same among different channels. For ease of analysis, we assume that there are 2 types of channels with datarate R_1 and R_2 available for SUs respectively, each type having M channels. Thus the total number of channels is $2M$. Table I gives the parameters used in the aggregate datarate analysis.

Assume that in different nodes, the average data flow length generated *in bytes* is the same. Assume that the data flow length, which are integer multiples of the time slot length follows independent geometrical distribution. Since there are 2 type of channels with different datarates, different channel

TABLE I
PARAMETERS FOR PERFORMANCE ANALYSIS.

Notation	Parameters Description
$2M$	The number of channels in 2 types; M channels for R_1 and R_2 respectively.
N	The number of SUs.
N_r	The total number of SUs that is ready to transmit or receive at the beginning of the tth time slot for all channels.
u_i	The number of SU pairs that successfully negotiate in tth time slot in channel type i, $i = 1, 2$.
v_i	The number of communicating pairs of SUs that finish data exchange at $(t-1)$th time slot in channel type i, $i = 1, 2$ and become ready at the beginning of the tth time slot.
c_i	The number of channels which have at least 1 potential receiver in the tth time slot in channel type i, $i = 1, 2$.
e_i	The number of idle channels in the tth time slot in channel type i, $i = 1, 2$.
d_i	The number of channels that are idle and have at least 1 potential receiver in them in the tth time slot in channel type i, $i = 1, 2$.
k_i	The number of communicating pairs in the $(t-1)$th time slot in channel type i, $i = 1, 2$.
m_i	The number of communicating pairs in the tth time slot in channel type i, $i = 1, 2$.
w	The number of SUs that have data to send in the tth time slot.
λ	The probability that an idle SU generates data flow.
μ_i	The probability that a pair of SUs finish data exchange and release the channel in channel type i, $i = 1, 2$.
γ	The probability that the PUs do not use the channels.

datarates will introduce different data flow length in number of time slots, i.e., different value of μ in geometrical distribution, denoted as μ_1 and μ_2. The probability of the length L_i of a data flow in time slots can therefore be expressed as $P(L_i = l_i)$=$\mu_i(1-\mu_i)^{l_i-1}$, $i = 1, 2$ for channel type 1 and 2 respectively. Since a data flow is transmitted in the same channel, it has the same μ during its transmission, no matter how many slots it takes.

Denote the switching penalty as T_{sw}. The switching penalty happens only at the first time slot of a successful communication session. Therefore, the average switching penalty with the number of time slots that a data transmission uses in channel type 1 and 2 is adopted, as $\overline{T}^i_{sw} = T_{sw}/\overline{L_i}$, where $\overline{L_i}$ is the average number of slots that a data flow transmission takes in channel type i, $i = 1, 2$. Denote the datarate, the length of time slot, and the length of quiet period by R_i, T_s, and T_q. The average flow length in bytes can be presented by $(T_s - T_q - \overline{T}^i_{sw}) \cdot R_i/\mu_i$, where $i = 1, 2$. Given $T_s >> \overline{T_q}$ and $T_s >> \overline{T}^i_{sw}$, for the same average length of data flow in bytes, we can ignore $\overline{T}^i_{sw}$ and approximately get that $R_i/R_j = \mu_i/\mu_j$, $\forall \mu \leq 1$.

Based on the above discussions, at any time slot, the system state can be presented by the number of communicating pairs of SUs in 2 kinds of channel types, i.e., (P_1, P_2). We can use a discrete-time Markov chain to analyze the aggregate datarate. State transfer happens when at least one communicating pair finishes transmission or a pair begins to transmit in either of these 2 channel types. Figure 4 presents a Markov chain in the case that there are 2 types of channels and each type has only 1 channel in it. The first element of the tuple in each state presents the number of communicating pairs in channel type 1 and the second element presents that in channel type 2. For example, state 10 means that there is one communicating pair in channel type 1 and no communicating pair in channel type 2. As there is only one channel in each type, the number of each element is up to 1 which means that there are in total 4 states. It is easy to extend it to 2 types of channels with several channels in each type and the difference is that the number of states of the Markov chain will be much larger.

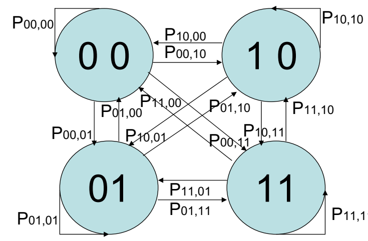

Fig. 4. A Markov chain model for aggregate datarate analysis.

In the following subsections, we will deduce first the state transfer probability of the Markov chain from $t-1$ to t, i.e., $P(m_1, m_2|k_1, k_2)$ and then get the steady state probability $\pi_{i,j}$, where $i, j \in [0, M]$. Finally, based on the probabilities obtained, the aggregate datarate will be calculated.

A. State transition probability

Given the number k_1 of communicating pairs in the $(t-1)$th time slot in channel type 1, the number v_1 of communicating pairs that become ready at the beginning of tth time slot follows binomial distribution, i.e., $P(v_1|k_1) = \binom{k_1}{v_1}(\mu_1)^{v_1}(1-\mu_1)^{k_1-v_1}$, $0 \leq v_1 \leq k_1$. The expression is similar for channel type 2. Then the number of nodes that is ready to transmit or receive at the beginning of the tth time slot N_r is: $N_r = N - 2(k_1 - v_1) - 2(k_2 - v_2)$, $0 \leq k_1, k_2, \leq \phi$, $\phi = min(M, N/2)$. The probability that w number of SUs have data to send at the tth time slot can be presented as $P(w|k_1, v_1, k_2, v_2) = \binom{N_r}{w}\lambda^w(1-\lambda)^{(N_r-w)}$, where $0 \leq w \leq N_r$. The number of idle SUs which are ready to receive data, denoted by potential receiver X_r, is $X_r = N_r - w$. Statistically, the idle SUs in channel type 1 and 2 denoted as X_{r1} and X_{r2} will be $X_{r1} = ||R_1/(R_1 + R_2) \cdot X_r||$ and $X_{r2} = X_r - X_{r1}$.

Denote by $P(c_1|k_1, v_1, k_2, v_2, X_{r1})$ the conditional probability of c_1 number of channels into which at least one potential receiver will jump at tth time slot, given that there are X_{r1} SUs potential receivers in channel type 1. This is analogous to put X_{r1} balls into M urns and then get the probability that there are c_1 urns that are not empty. The possible solution can be find in reference [10] even though there is a slight difference[1].

[1]In [10], the calculation of channel with exactly one transmitter is considered. In our analysis, we consider the channels with one or more available potential receivers, which include the situation that several transmitters may contend for channel access at the same time slot on the same channel. The successful communication pair will still be only one after the negotiation process.

As the probability of c_1 is not correlated to k_1,v_1,k_2,v_2, given X_{r1}, we get $P(c_1|k_1, v_1, k_2, v_2, X_{r1}) = P(c_1|X_{r1})$. The same result applies to channel type 2.

Denote by $P(e_1|k_1, v_1, k_2, v_2, X_{r1}, c_1)$ the probability that there are e_1 number of the idle channels of channel type 1, given that there are k_1 communicating pairs in $(t-1)$th time slot and v_1 pairs of SUs that have finished communications at the end of $(t-1)$th time slot. Then,

$$\begin{aligned} &P(e_1|k_1, v_1, k_2, v_2, X_{r1}, c_1) = P(e_1|k_1, v_1) \\ &= \tbinom{M-k_1+v_1}{e_1}\gamma^{e_1}(1-\gamma)^{M-k_1+v_1-e_1}. \end{aligned} \tag{1}$$

Denote by $P(d_1|k_1, v_1, k_2, v_2, X_{r1}, c_1, e_1)$ the conditional probability that d_1 number of the channels that are idle and have at least one potential receiver, given e_1 idle channels and c_1 channels that have at least one potential receiver in channel type 1. According to the hypergeometric distribution [7], [10], we obtain

$$\begin{aligned} &P(d_1|k_1, v_1, k_2, v_2, X_{r1}, c_1, e_1) \\ &= (d_1|X_{r1}, c_1, e_1) = \tbinom{e_1}{d_1}\tbinom{M-e_1}{c_1-d_1}/\tbinom{M}{M-c_1}, \end{aligned} \tag{2}$$

where $0 \leq d_1 \leq c_1$.

For channel type 1, combining the above 2 equations, we get

$$\begin{aligned} &P(d_1|k_1, v_1, k_2, v_2, X_{r1}, c_1) = P(d_1|k_1, v_1, X_{r1}, c_1) \\ &= \sum_{e_1=0}^{M-k_1+v_1} P(d_1|X_{r1}, c_1, e_1)P(e_1|k_1, v_1) \\ &= \sum_{e_1=0}^{M-k_1+v_1} \frac{\tbinom{e_1}{d_1}\tbinom{M-e_1}{c_1-d_1}}{\tbinom{M}{M-c_1}} \tbinom{M-k_1+v_1}{e_1}\gamma^{e_1}(1-\gamma)^{M-k_1+v_1-e_1}. \end{aligned} \tag{3}$$

We approximate the probability that a receiver has data flow to be sent by a transmitter with $w/(N-1)$ [7]. Then we can approximately[2] calculate the probability that u_1 number of the SUs pairs that successfully negotiate in these d_1 channels at the tth time slot [10], $P(u_1|k_1, v_1, k_2, v_2, w, c_1, d_1)$, as

$$\begin{aligned} &P(u_1|k_1, v_1, k_2, v_2, w, c_1, d_1) = P(u_1|k_1, v_1, w, d_1) \\ &= \tbinom{d_1}{u_1}\left(\frac{w}{N-1}\right)^{u_1}\left(1-\frac{w}{N-1}\right)^{d_1-u_1}. \end{aligned} \tag{4}$$

Because $u_1 = m_1 - (k_1 - v_1)$, we can give the probability

$$\begin{aligned} &P(m_1|k_1, v_1, w, c_1, d_1) \\ &= \tbinom{d_1}{m_1-(k_1-v_1)}\left(\frac{w}{N-1}\right)^{m_1-(k_1-v_1)}\left(1-\frac{w}{N-1}\right)^{d_1-(m_1-(k_1-v_1))}. \end{aligned} \tag{5}$$

For channel type 1, by using the $P(m_1|k_1, v_1, w, c_1, d_1)$, $P(d_1|k_1, v_1, X_{r1}, c_1)$, $P(c_1|X_{r1})$, we can obtain that

$$\begin{aligned} &P(m_1|k_1, v_1, w, Xr_1) \\ &= \sum_{c_1=0}^{M}\sum_{d_1=0}^{c_1} P(m_1|k_1, v_1, w, Xr_1, c_1, d_1)\times \\ &P(d_1|k_1, v_1, w, X_{r1}, c_1)P(c_1|X_{r1}). \end{aligned} \tag{6}$$

Similar expressions for Eqs. (1)-(6) can be easily found for channel type 2.

[2]For simplicity, we approximate that the utilization probability of idle channels with more than one potential receiver is the same as the case with only one potential receiver in the analysis, since differentiating channels according to the number of potential receivers will introduce extreme complexity in the analysis. However, we are aware of that it is less likely that several intended receivers will be unavailable at the same time in practice.

Note that $P(m_1|k_1, v_1, w, Xr_1)$ and $P(m_2|k_2, v_2, w, Xr_2)$ are probabilities analyzed in different types of channels and they are independent. Thus the joint probability can be expressed as

$$\begin{aligned} &P(m_1, m_2|k_1, v_1, k_2, v_2, w, Xr_1, Xr_2) \\ &= P(m_1|k_1, v_1, w, Xr_1)\cdot P(m_2|k_2, v_2, w, Xr_2). \end{aligned} \tag{7}$$

With our hopping sequence adjustment method, statistically, the probability of Xr_1 and Xr_2 can be expressed as

$$\begin{aligned} &P(X_{r1} = j, X_{r2} = N_r - w - j) \\ &= \tbinom{N_r-w}{j}(R_1/(R_1+R_2))^j(R_2/(R_1+R_2))^{N_r-w-j}. \end{aligned} \tag{8}$$

Then, we can obtain

$$\begin{aligned} &P(m_1, m_2|k_1, v_1, k_2, v_2, w) \\ &= \sum_{j=0}^{N_r-w} P(m_1, m_2|k_1, v_1, k_2, v_2, w, Xr_1, Xr_2)\times \\ &P(X_{r1} = j, X_{r2} = N_r - w - j). \end{aligned} \tag{9}$$

It is obviously that $P(v_1|k_1)$ and $P(v_2|k_2)$ are independent, then it is found that

$$P(v_1, v_2|k_1, k_2) = P(v_1|k_1)P(v_2|k_2). \tag{10}$$

With the help of $P(w|k_1, v_1, k_2, v_2)$, we can finally compute

$$\begin{aligned} &P(m_1, m_2|k_1, k_2) \\ &= \sum_{v_1=0}^{k_1}\sum_{v_2=0}^{k_2}\sum_{w=0}^{N_r} P(m_1, m_2|k_1, v_1, k_2, v_2, w)\times \\ &P(w|k_1, v_1, k_2, v_2)P(v_1, v_2|k_1, k_2). \end{aligned} \tag{11}$$

B. Steady-state probability

Known the transition probabilities, we can calculate the probability for steady-state of the Markov chain. The steady-state probability is given by

$$\Pi = \Pi\mathbf{P}, \tag{12}$$

where Π is a row vector whose elements, $\pi_{i,j}$, sum to 1 as shown in Eq. (13), and $\pi_{i,j}$ is the steady-state probability with i and j communicating pairs in channel type 1 and 2 respectively. P is the transition matrix, formed by $P(m_1, m_2|k_1, k_2)$, as

$$\mathbf{P} = \begin{bmatrix} P(0,0|0,0) & P(0,1|0,0) & \cdots & P(M,M|0,0) \\ P(0,0|0,1) & P(0,1|0,1) & \cdots & P(M,M|0,1) \\ \vdots & \vdots & \vdots & \vdots \\ P(0,0|M,M) & P(0,1|M,M) & \cdots & P(M,M|M,M) \end{bmatrix},$$

The sum of all probabilities would be unity, as

$$\sum_{i,j}\pi_{i,j} = 1. \tag{13}$$

By solving Eqs. (12) and (13), we can find all steady-state probabilities, $\pi_{i,j}$, for $0 \leq i, j \leq M$.

If the Markov chain is irreducible and aperiodic, then there is a unique stationary distribution. In this case, $\mathbf{P}^{\kappa}$ converges to a rank-one matrix in which each row is the steady distribution Π, i.e., $lim_{\kappa\longrightarrow\infty}\mathbf{P}^{\kappa} = \mathbf{E}\Pi$, where $\mathbf{E}$ is the column vector with all entries equaling to 1 and κ is the exponent of P. This character of the Markov chain can be used to verify the validity of our analysis[3].

[3]Indeed, we calculated $lim_{\kappa\longrightarrow\infty}\mathbf{P}^{\kappa}$ and find that it converges to Π and $\sum_{i,j}\pi_{i,j} = 1$ from the numerical results. The validity of the analysis is therefore verified.

C. Aggregate datarate

The transmissions that are not finished in $(t-1)$th time slot will be buffered in the tth time slot in the presence of PUs. Denote $\overline{N_{t1}}(k_1, v_1, \gamma)$ as the average number of ongoing communicating pairs of SUs that exchange data in tth time slot in channel type 1 [10],

$$\overline{N_{t1}}(k_1, v_1, \gamma) = \sum_{i=0}^{k_1-v_1} i\binom{k_1-v_1}{i}\gamma^i(1-\gamma)^{k_1-v_1-i}. \quad (14)$$

Then the aggregate datarate, denoted as S which is the sum of data transmitted over channel type 1 and 2, denoted as S_1 and S_2, can be expressed as:

$$S = S_1 + S_2. \quad (15)$$

where

$$S_1 = (T_s - \overline{T}_{sw}^1 - T_q) \cdot R_1/T_s \times \sum_{k_1=0}^{\phi}\sum_{k_2=0}^{\phi}\sum_{m_1=0}^{\phi}\sum_{m_2=0}^{\phi}\sum_{v_1=0}^{k_1}\sum_{v_2=0}^{k_2} P(k_1, m_1, v_1, k_2, m_2, v_2)\times [\overline{N_{t1}} + m_1 - (k_1 - v_1)], \quad (16)$$

and

$$\begin{aligned} P(k_1, m_1, v_1, k_2, m_2, v_2) &= P(m_1, v_1, m_2, v_2|k_1 k_2)\pi_{k_1,k_2} \\ &= P(v_1, v_2|k_1, k_2, m_1, m_2)P(m_1, m_2|k_1 k_2)\pi_{k_1,k_2} \\ &= P(v_1, v_2|k_1, k_2)P(m_1, m_2|k_1 k_2)\pi_{k_1,k_2}. \quad (17) \end{aligned}$$

Similar expressions can be found for S_2 from Eqs. (14), (16) and (17).

The above analysis result can also be extended to a more general case where there are more than two types of channels. Denote N_c as the number of channel types. We can form a Markov chain with N_c elements and each element stands for the number of communicating pairs in channels with the same datarate. In this case, Eq. (8) should be revised as a multinomial distribution instead of binomial distribution, as shown in Eq. (18).

$$P(X_{r_1} = x_{r_1}, X_{r_2} = x_{r_2}, \cdots X_{r_{N_c}} = x_{r_{N_c}}) = \begin{cases} \frac{(N_r - w)!}{x_{r_1}! \cdots X_{r_{N_c}}!} \quad (\frac{R_1}{R_1 + \cdots R_{N_c}})^{x_{r_1}} \cdots (\frac{R_{N_c}}{R_1 + \cdots R_{N_c}})^{x_{r_{N_c}}}, \\ \quad when \sum_{i=1}^{N_c} x_{r_i} = N_r - w. \\ 0, \quad otherwise. \end{cases} \quad (18)$$

Correspondingly, Eq. (9) can be expressed as:

$$\begin{aligned} &P(m_1, \cdots m_{N_c}|k_1, v_1, \cdots k_{N_c}, v_{N_c}, w) \quad (19) \\ &= \sum_{\sum_{i=1}^{N_c} x_{r_i} = N_r - w} P(X_{r_1} = x_{r_1}, X_{r_2} = x_{r_2}, \cdots X_{r_{N_c}} = x_{r_{N_c}}) \\ &\times P(m_1, \cdots m_{N_c}|k_1, v_1, \cdots k_{N_c}, v_{N_c}, w, X_{r_1}, \cdots X_{r_{N_c}}). \end{aligned}$$

Other part of the analysis when there are more than two types of channels is quite similar to that of two types of channels. With the analysis of probability, we can find the steady state of Markov chain and finally get the aggregate datarate in this more complicated case.

V. Numerical results and discussions

In this section, based on the analytical results obtained in Section IV, we illustrate the numerical results of the proposed protocol and compare its performance with that of DRI-MAC. The results of DRI-MAC are obtained given that nodes have equal chance to access these two types of channels.

A. Parameter configuration

We assume that the hop sequence adjustment information is known ideally by SUs. The parameters used to calculate aggregate datarate are configured as follows: $T_s = 1000\ \mu s$, $T_q = 10\ \mu s$, $T_{sw} = 100\ \mu s$, $R_1 = 2\ Mbps$, and $R_2 = 10\ Mbps$. With this time slot and datarate configuration, it is enough to finish a negotiation within a small portion of a time slot [7] and we have also $T_s >> T_q$ and $T_s >> \overline{T}_{sw}^i$, which are in accordance with the discussions in Section IV.

B. Aggregate datarate as a function of λ

We start our performance evaluation by considering the aggregate datarate as a function of λ. To do so, we set other parameters, i.e., the total number of SUs as $N = 20$, channel occupancy by PUs as $\gamma = 0.7$, and transmission completion rate at two channels as $\mu_1 = 0.05$ and $\mu_2 = 0.25$ respectively. With these parameter settings, we can estimate that the average data flow length is $2Mbp * (1000\mu s - 10\mu s - 100\mu s/20)/0.05/8 \approx 5KB$. This implies that the time slots needed for transmitting this data flow are respectively 20 slots at R_1 and 4 slots at R_2, on average.

Fig. 5 depicts the obtained aggregate datarate according to λ by using the DRA-MAC and DRI-MAC protocols, where the number of channels at each datarate is set as $M = 3$ and $M = 4$ respectively. From this figure, we can see that the achieved aggregate datarate is 0 when $\lambda = 0$ or 1. This is because that when $\lambda = 0$, there is no transmitter and in the case of $\lambda = 1$, there are no receivers. When $\lambda = 1$, all SUs have data to transmit. SUs will leave their own channel and come to the intended receivers' channel for communication. In this case, theoretically, every SU deviates from its hopping sequence denoted channel thus these SUs cannot find each other. When λ is small, SUs do not generate many data flows. This means that the totally generated traffic load by SUs is so light that it does not even saturate the channels that have the lower datarate. As a result, the aggregate datarate difference between these two MAC protocols is not significant in this case, with both $M = 4$ and $M = 3$. However, when the traffic load becomes heavier and idle SUs nodes have more data flows to transmit, the advantage of the proposed protocol is evident. As shown in Fig. 5, over a wide range of λ, significant aggregate datarate improvement has been achieved by DRA-MAC, compared with what is obtained by its counterpart, DRI-MAC. For example, at $\lambda = 0.5$, DRA-MAC reaches aggregate datarate of 16 $Mbps$ while 14 $Mbps$ is obtained by DRI-MAC, which means that an improvement of 14% has been achieved.

Comparing the difference between $M = 3$ and $M = 4$, we can observe that when the channel number is larger, the enhancement is more significant. This is because that when M is greater, more channels with high datarates are available for SU nodes. With our proposed method, SUs get better chance to transfer their data flows over the higher datarate channel, leading to an increased aggregate datarate.

Note also that in [10], the peak value of aggregate datarate is achieved around $\lambda = 0.25$ and the aggregate datarate becomes lower when λ gets larger. It is because that in

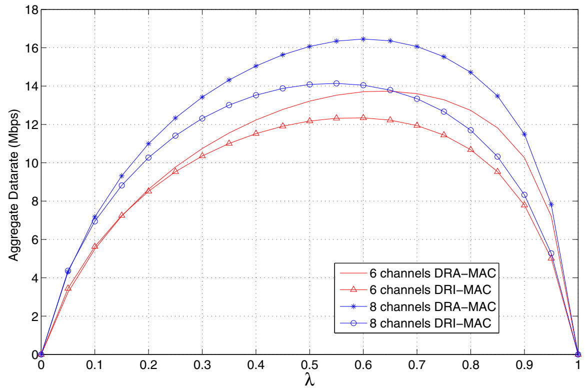

Fig. 5. Aggregate datarate comparison of DRA-MAC and DRI-MAC as a function of λ.

[10] it calculates the channels with exact one transmitter in $P(c_1|k_1, v_1, k_1, v_1, X_{r1})$. When the sending probability (λ) becomes larger, the probability of channels with exact one transmitter will be lower. Consequently, the aggregate datarate is lower. In contrast, in our scheme, we consider the channel with one or more potential receivers (see footnote 1), which means that the number of channels that has two or more transmitters are also counted in, because after negotiation these channels can also be used. Consequently, the DRI-MAC curves shown in Fig. 5 are also obtained considering one or more receivers. Therefore, the peak value is obtained when λ is around 0.55 for DRI-MAC.

C. Impact on aggregate datarate by the number of SUs

In this subsection, we continue our performance evaluation by varying the number of SUs in the system. Now the other parameters are fixed as $\lambda = 0.7$, $\gamma = 0.7$, $\mu_1 = 0.05$, and $\mu_2 = 0.25$, while N is a variable. The aggregate datarate of DRA-MAC versus DRI-MAC as the number of nodes N varies is illustrated in Fig. 6.

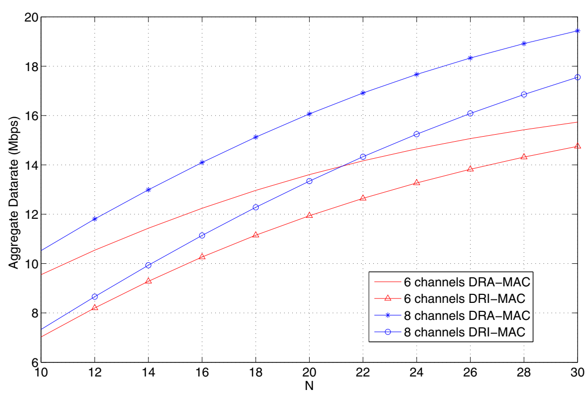

Fig. 6. Aggregate datarate comparison of DRA-MAC and DRI-MAC as a function of N.

In Fig. 6, again, DRA-MAC outperforms DRI-MAC for all ranges of investigated values. This is because that more nodes jump to the higher datarate channels according to the proportion of datarates in 2 types of channels rather than uniform hopping sequences, leading to higher aggregate datarates. Interestingly in this case, larger differences are observed when N is smaller, with both $M = 3$ and $M = 4$. It is because that when the number of SU nodes is smaller, the system is far from saturation. At the same time, idle SUs nodes have many data flows to send since $\lambda = 0.7$ which indicates a high transmission probability for SUs. Once one communication pair is re-allocated from the low datarate channel to high datarate channel, it contributes more to the achieved aggregate datarate. For instance, assume that there are four ongoing data flows in the system, two at each type. If one of the two low datarate flows is re-allocated to the high datarate channel, the aggregate datarate will be significantly increased since we have now 3 out 4 flows using the high datarate channel. When the number of SUs gets larger, the probability that more channels are occupied by communicating pairs will be higher. In other words, with a large N, the channels are close to saturation and there is less room for aggregate datarate improvement, no matter how you balance the hop sequences of the SU nodes. This explains why the difference between the two methods becomes smaller as N increases.

D. Impact on aggregate datarate by channel datarate

Finally, we investigate the system performance by adjusting the channel datarate of one of these two channels. In this investigation, we set the other parameters fixed as $R_1 = 2$ $Mbps$, $\lambda = 0.7$, $\gamma = 0.7$, and $N = 20$, while R_2 is varying. In order to ensure the average length of data flows *in bytes* in different channels are the same, μ_1 is fixed as 0.05 while μ_2 is 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 when R_2 equals to 2, 4, 6, 8, 10, 12 $Mbps$ respectively.

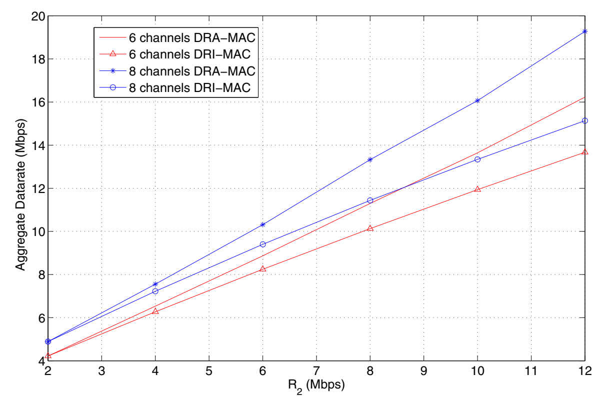

Fig. 7. Aggregate datarate comparison of DRA-MAC and DRI-MAC as R_2 varies.

Fig. 7 illustrates the aggregate datarate achieved by DRA-MAC and DRI-MAC respectively. From this figure we can conclude that the improvement of the proposed method is more significant when the datarate of R_2 increases. This is because that with a larger R_2, higher aggregate datarate is achieved every time when an SU pair is re-allocated from an R_1 channel to an R_2 channel. Note that when $R_2 = R_1$, the aggregate datarate achieved by both methods is the same because in this case, and the hops according to the proposed MAC is also uniformly distributed, which implies that DRI-MAC is actually a special case of DRA-MAC.

From this figure, we can also observe that when R_2 is not 2 times higher than R_1, the improvement is not so evident. Considering the fact that extra beacon overhead is needed for DRA-MAC, one would probably prefer to adopt DRI-MAC when if R_2 is not more than twice as high as R_1, since DRI-MAC is less complicated.

E. Extra overhead estimation of DRA-MAC

Now we approximately calculate the extra overhead introduced by DRA-MAC due to the required dissemination of the hop sequence adjustment information. Assume that there are 20 nodes, 4 channels with 2 $Mbps$ and 4 channels with 10 $Mbps$, the hopping period is 128 hops and beacon interval is 5 seconds. We can get the average overhead as 128*3*1*20/5+(128*3+48)*8*20/5=15.36 $Kbps$, where 128 means that there are 128 hops, 3 means that 8 channels can be presented in 3 bits. The calculation has 2 parts. The first part presents the beacons that are broadcast by the node itself. The second part presents the beacons that are rebroadcast by other nodes. 8 is an estimation of the number of re-broadcasting beacon packets because there are 8 channels in total with $M = 4$ and one packet can be forwarded in each channel. Since the other nodes that re-broadcast the beacon have to attach the MAC of the original node, it has an extra 48 bits due to the length of a MAC address.

From the above estimation, we can conclude that the extra overhead introduced by DRA-MAC is pretty small. This means that the additional protocol cost by the proposed MAC, typically in the order of a few Kbps, in order to achieve possibly a few Mbps aggregate datarate improvement. Anyhow, it is beneficial to consider this effect for our protocol design, so that further improvement can be achieved.

VI. Conclusions

In this paper we have proposed a channel-hopping based parallel rendezvous datarate-aware MAC protocol for multi-rate multi-channel cognitive radio networks equipped with one transceiver. With this protocol, each SU can adjust their hopping sequences according to the datarates in different channels to enhance the aggregate datarate, based on its basic hopping sequence. A mathematical model has been developed to analyze the performance of the proposed MAC protocols. Numerical results and comparison between DRA-MAC and DRI-MAC show that our proposed protocol generally outperforms the existing one. The improvement compared with DRI-MAC is more significant when more channels are available for SUs, fewer SUs are in the network and the difference between high datarate and low datarate is larger.

Appendix

Proposition: Let φ_i be the likelihood of an SU that will jump into channel i after using the proposed method in Section III. B. For every channel i, φ_1:$\varphi_2\cdots$ φ_G=R_1:$R_2\cdots R_G$.

Proof: Since nodes jump according to the uniformly generated sequence before adjustment, the probability that an SU jumps to channel $i, 1 \leq i \leq G$, is equal. Let us arrange the set of G channels according to the value of R_j as $\{1, 2, \cdots, l, l+1, \cdots, G\}$ such that $R_j \leq \overline{R} \Leftrightarrow j \leq l$ and $R_j \leq R_i \Leftrightarrow j < i$. Let $B \in \{channel\ j | R_j < \overline{R}, j = 1, 2, \cdots G\}$. After the adjustment, we can see that

$$\varphi_{1,2\cdots l} = R_{1,2\cdots l}/\overline{R} \ \ and$$

$$\varphi_{l+1,l+2\cdots G} = 1 + \frac{\sum_{i\in B}(1 - R_i/\overline{R}) \cdot (R_{l+1,l+2,\cdots G} - \overline{R})}{\sum_{k\in A}(R_k - \overline{R})}.$$

In order to keep φ_1:$\varphi_2\cdots$ φ_G=R_1:$R_2\cdots R_G$, we should prove that

$$1 + \frac{\sum_{i\in B}(1 - R_i/\overline{R}) \cdot (R_{l+1,l+1,\cdots G} - \overline{R})}{\sum_{k\in A}(R_k - \overline{R})} = R_{l+1,l+2\cdots G}/\overline{R}.$$

When $j > l$, we can observe that

$$\begin{aligned}
&1 + \sum_{i\in B}(1 - R_i/\overline{R}) \cdot (R_j - \overline{R}) / \sum_{k\in A}(R_k - \overline{R}) \\
&= \frac{\sum_{i\in B}(\overline{R} - R_i)}{\sum_{k\in A}(R_k - \overline{R})} \cdot \frac{R_j}{\overline{R}} + 1 - \frac{\sum_{i\in B}(\overline{R} - R_i)}{\sum_{k\in A}(R_k - \overline{R})} \\
&= \frac{\sum_{i\in B}(\overline{R} - R_i)}{\sum_{k\in A}(R_k - \overline{R})} \cdot \frac{R_j}{\overline{R}} + \frac{\sum_{k\in A}(R_k - \overline{R}) + \sum_{i\in B}(R_i - \overline{R})}{\sum_{k\in A}(R_k - \overline{R})} \\
&= \frac{\sum_{i\in B}(\overline{R} - R_i)}{\sum_{k\in A}(R_k - \overline{R})} \cdot \frac{R_j}{\overline{R}} = \frac{R_j}{\overline{R}}.
\end{aligned}$$

Now we can conclude that:
φ_1:$\varphi_2\cdots\varphi_G$=$R_1/\overline{R}$:$R_2/\overline{R}\cdots R_G/\overline{R}$ =R_1:$R_2\cdots R_G$. ■

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